A Reinterpretation of the Odar and Hamilton Data on the Unsteady Equation of Motion of Particles

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The experimental data and correlations derived by Odar and Hamilton have been the basis of studies that included the history term in the expression of the unsteady drag. Recent studies have shown that the value of the added mass coefficient is constant and equal to ½ over a very large range of Reynolds numbers. Recent studies on the history term have proven that its form is not correct at high Re. However, the experimental data are accurate at low Re and, most probably, they represent the most reliable set of experimental data on the unsteady force on solid spheres. We conducted a study to re-calculate the functional form of the history term in the unsteady equation of motion at low Re and to derive a new correlation for the so-called "history force coefficient," Δ_H. The new correlation is expressed in terms of the Reynolds and Strouhal numbers of the particle. © 2010 American Institute of Chemical Engineers *AIChE J,* 57: 2997–3002, 2011
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Introduction

Several practical applications ranging from coal and droplet combustion in boilers and burners to the transport of aerosols in the atmosphere involve the unsteady motion of solid and fluid particles. Boussinesq[1] and Basset[2] first derived the unsteady equation of motion of a small spherical solid particle at $Re \ll 1$ (creeping flow conditions), which in terms of the drag force exerted by the fluid on the particle, F_i, may be written as follows:

$$F_i = 6\pi R\mu(u_i - v_i) + \left(\frac{4}{3}\pi R^3\rho\right)\frac{d(u_i - v_i)}{dt} + R^2(\pi\rho\mu)^{\frac{1}{2}}\int_{t_0}^{t}\frac{\frac{d(u_i-v_i)}{dt'}}{\sqrt{t-t'}}dt' \quad (1)$$

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where F_i is the total force acting on the particle; v_i and u_i are the velocities of the particle and the fluid respectively; ρ the density of the fluid; t is the time; R is the radius of the sphere; and μ is the viscosity of the fluid. The first term in Eq. 1 represents the friction drag on the particle. The second term is called "added mass" or "virtual mass" and accounts for the fluid that is accelerated or decelerated with the particle. The last term represents the cumulative effect of the unsteady motion of the particle on the fluid velocity field and is called the "history term" or the "Basset term." Equation 1 is often times referred to as the "Boussinesq-Basset expression."

Because a great deal of the practical applications of gas–solid flows occur at higher Reynolds numbers, several researchers tried to extend this expression to higher Reynolds numbers. Expressions for the steady drag term at higher Re were derived from experimental data, as for example, the expression by Schiller and Nauman,[3] which is still used today in steady-state applications. Regarding the transient terms of the unsteady equation of motion for hydrodynamic force, Odar and Hamilton[4] (OH-64) conducted an extensive set of experiments and were the first to propose empirical coefficients for all the terms of Eq. 1, which would make the equation valid at higher Re. The experiments were conducted with a 2.5" sphere that oscillated in a tank filled with oil at stagnant conditions. The Reynolds numbers of the original experiments were in the range 0–62. OH-64 proposed the following functional form for the unsteady drag force acting on a spherical particle:

$$F_i = C_D \pi R^2 \frac{1}{2} \rho (u_i - v_i)|u_i - v_i| + C_A \left(\frac{4}{3}\pi R^3 \rho\right) \frac{d(u_i - v_i)}{dt} + C_H R^2 (\pi \rho \mu)^{\frac{1}{2}} \int_{t_0}^{t} \frac{\frac{d(u_i - v_i)}{dt'}}{\sqrt{t - t'}} dt' \quad (2)$$

where C_D, C_A, and C_H are three empirical coefficients, which were given by correlation functions, derived from experimental data. The product $(u_i - v_i)|u_i - v_i|$ in the steady drag coefficient ensures that this term has the same sign as the relative velocity of the particle $(u_i - v_i)$. It must be noted that more recent research (Auton et al.[5]) has shown that the fluid velocity derivative in the added mass term at the right-hand side of the equation is the substantial derivative (Du_i/Dt). In the original experiment by OH-64 the fluid is stagnant and this substantial derivative vanishes. Equation 2 represents the expression as it was given in the original paper. The revised equation that was derived in this study (Eq. 13 below) includes this correction for the added mass term.

Of the three coefficients, C_D is the steady-state drag coefficient. OH-64 used the Schiller and Nauman[3] correlation for the steady-state drag coefficient:

$$C_D = \frac{24}{\mathrm{Re}}\left[1 + 0.15\mathrm{Re}^{0.687}\right] \quad (3)$$

where the Reynolds number is defined as:

$$\mathrm{Re} = \frac{2\rho|u_i - v_i|R}{\mu} \quad (4)$$

The other functions, C_A and C_H, were obtained by correlating the experimental data of OH-64. The correlations proposed are as follows:

$$C_A = 1.05 - \frac{0.066}{Ac^2 + 0.12} \quad (5)$$

and

$$C_H = 2.88 + \frac{3.12}{(Ac + 1)^3} \quad (6)$$

The acceleration number, Ac, which was chosen by OH-64 as the primary variable for the correlations is defined as follows:

$$Ac = \frac{(u_i - v_i)^2}{2R\left|\frac{d(u_i - v_i)}{dt}\right|} \quad (7)$$

Following the original study, the empirical equation and the coefficient correlations were verified in a subsequent paper by Odar[6] who used data for the total unsteady hydrodynamic force exerted by the fluid. Since their publication, the expressions for the unsteady hydrodynamic force on particles have been used in numerous analytical and computational studies. An extensive treatment on the transient equation of motion at creeping flow conditions as well as at finite Reynolds numbers, which includes several more recent studies that have verified the OH-64 correlations for the total hydrodynamic force, may be found in a review article by Michaelides[7] and the monograph by Michaelides.[8]

Regarding the development of the transient equation of motion of spheres at creeping flow conditions, Faxen[9] established the influence of the proximity of boundaries and of non-uniform flow around the sphere. Maxey and Riley[10] used modern analytical techniques and derived the transient, creeping flow equation for a sphere that includes the history as well as the Faxen terms in the hydrodynamic drag. The OH-64 study was conducted before the study by Maxey and Riley[10] and used simply the original formulation of the transient equation of a sphere as presented by Basset.[2] It is rather fortuitous for the OH-64 data, however, that the conditions used in the OH-64 study are such that the Faxen term contribution to the total hydrodynamic drag are very small (<1%). Michaelides[11] derived an explicit form of the transient equation of motion by transforming the original implicit first-order integro-differential equation to a second-order equation.

At higher Reynolds numbers, Sano[12] showed analytically that the history term decays faster than $t^{-1/2}$ in the impulsive motion of the sphere. Later, Lovalenti and Brady[13] derived a general expression for the history term at finite but small Reynolds numbers under any type of motion, while Mei and Adrian[14] and Mei[15] derived an analytical expression of the history term during the oscillatory motion of a small particle. Chang and Maxey[16] showed numerically that, during the initial stages of its motion, the history term of a spherical particle follows the $t^{-1/2}$ decay as shown in Eq. 1 even at significantly higher Reynolds numbers. Finally, Kim et al.[17] proposed a more general convolution integral for this term. It must be pointed out, however, that all these recent studies are analytical or numerical and that, up to-date, the OH-64 data remain the only set of detailed experimental data that calculated the history term of the unsteady equation of motion of a solid sphere. An extensive review of the all the developments on the transient equation of motion and heat transfer from particles at creeping flow conditions as well as at higher Reynolds numbers may be found in the review article by Michaelides[7] and the monograph by Michaelides.[8]

The Need to Reinterpret the Odar and Hamilton Data

Regardless of the results of analytical studies that focused on the decay of the transient terms, the general practice of using the OH-64 equation, that is Eq. 2, to describe the unsteady motion of spherical particles was deemed successful in the past and has been verified by several other studies as, for example, by Tsuji et al.[18] The calculations emanating from this expression showed good agreement with sets of experimental data. However, all these studies used and verified the total hydrodynamic force and not the three separate parts of it. Thus, the use of the OH-64 expression became popular in engineering calculations since the 1970s because it yielded accurate results for the total hydrodynamic force, F_i. The reason for this manifested accuracy and the popularity of the transient expression is the relative ease of calculations with the aid of computers and the close agreement of the results with experimental data. The agreement with experimental data is rather fortuitous and due to the following reasons:

(a) The equation has a sound experimental basis, since it is a correlation of accurate experimental results and

(b) The equation has been derived from measurements of the total hydrodynamic force and has always been used in calculations to determine, again, the total hydrodynamic force, and not any of its three parts, separately.

It must be noted that the experiments, from which the coefficients of the OH-64 expression as well as other semi-empirical expressions emerged, measured the total hydrodynamic force on the sphere under various flow conditions. Then, by a series of assumptions and deductions, the experimentalists estimated the three parts of the pertinent expressions and, hence, determined the three coefficients, C_D, C_A, and C_H, as functions of parameters such as Re or Ac. Similarly, for any verification of the derived expressions in subsequent studies, the total hydrodynamic force was always computed as a single entity. All comparisons were performed for the entire computed hydrodynamic force with sets of experimental data, which were similar to the ones from which the force was determined in the first place. Therefore, even if the calculations of the three parts of the hydrodynamic force, separately, were laden with error, these errors cancelled and the calculation of the sum of the terms, which is equal to the total hydrodynamic force, would have had very low error. Because of this coincidence in the ''determination'' and ''verification'' of the results, it would have been rather surprising if close agreement between ''experiments'' and ''theory'' were not obtained. This fortuitous agreement is the main reason why semi-empirical expressions, such as the one by OH-64, are still considered accurate enough and are being used extensively in the Lagrangian computations of particles during transient flow calculations. The derived results for the hydrodynamic force, as a whole, can be trusted within their range of applicability because they have a sound experimental basis.

Regarding the three separate terms in Eq. 2, several analytical and experimental results have shown that the added mass coefficient, C_A, in Eq. 2 is constant and equal to $1/2$. The origin of this term is from inviscid, potential flow theory as presented by Poisson[19] and there is no a priori reason for this term to be a function of the Reynolds number or the acceleration number, Ac. Several analytical studies in the past confirmed that C_A is actually constant for spheres. In addition, Bataille et al.[20] used the experimental technique of Darwin[21] and conducted a series of experiments, which showed unequivocally that the added mass coefficient is equal to $1/2$ in the range $0 < Re < 500$.

These considerations for the coefficient C_A, cast significant doubts that the two last terms of Eq. 2 accurately represent the added mass and the history term in the unsteady equation of motion. The coefficient of the first term, C_D, which represents the steady drag on the spherical particle was determined by independent experimental steady drag force data and is considered to be accurate. However, OH-64 measured the total transient hydrodynamic force in a series of accurate experiments. Then, they subtracted the steady drag using the drag coefficient as expressed in Eq. 3 and, calculated the sum of the history and added mass terms. Subsequently, using a series of inferences OH-64 separated the effect of the other two terms, and derived separate correlations for the coefficients C_A and C_H. Therefore, the sum of the last two terms in Eq. 2 stems from accurate experimental data, even though the inferences that gave rise to the correlations (5) and (6) may not be correct. If $C_A = 1/2$ for a wide range of Reynolds numbers, then correlation (6) must be re-evaluated to yield an accurate expression for the history term.

The study by OH-64 is considered to be the most detailed experimental study on the transient equation of motion or spherical particles. Despite the error in the separate determination of the added mass and the history terms, the experimental data themselves for the total hydrodynamic force, F_i, are accurate. Therefore, it is good to use this set of accurate and meticulously obtained data in order to obtain a more accurate representation of the history term. For this reason, we undertook a study to re-interpret the experimental data by OH-64 under the light of the more recent studies and the known, constant coefficient for the added mass term. From this re-interpretation of the data a new correlation was derived for the coefficient, C_H, of the history term.

Analysis of the Experimental Data

At first, Eq. 2 was cast in the correct functional form, that conforms with the subsequent research on the unsteady equation of motion: this form that uses the substantial derivative (D/Dt) for the fluid velocity in the added mass term and appears as follows:

$$F_i = \frac{1}{2} C_D \pi R^2 \rho |u_i - v_i| (u_i - v_i) + \frac{1}{2} \Delta_A \left(\frac{4}{3} \pi R^3 \rho \right) \left(\frac{Du_i}{Dt} - \frac{dv_i}{dt} \right) + \\ + \Delta_H R^2 (\pi \rho \mu)^{\frac{1}{2}} \int_{t_0}^{t} \frac{\frac{d(u_i - v_i)}{dt'}}{\sqrt{t - t'}} dt' \quad (8)$$

According to independent experimental data, an accurate expression for C_D is given by Eq. 3, whereas an accurate expression for Δ_A is $\Delta_A = 1$ (or $C_A = 1/2$) for the entire range of the acceleration and Reynolds numbers of the original data by Odar and Hamilton. This allowed us to use the entire set of the original data to determine the functional form of the parameter Δ_H in Eq. 8, or C_H in Eq. 2.

The original set of experimental data of the OH-64 study is not available. For this reason, we followed the original OH-64 study and ''back-tracked'' through the expressions they used to calculate their final correlations. The original study used a solid spherical particle in oscillatory motion inside a fluid at rest to measure the total transient hydrodynamic force. We followed the same method OH-64 used to reduce the original set of data and performed several numerical ''trials'' to compute the total hydrodynamic force on the oscillating sphere with a set of similar frequencies and motion amplitudes as in the original study. The parameters used in each trial are shown in Table 1, which follows.

The values of the fluid properties reported in the original study are: $\rho = 889.07$ kg/m^3 and $\mu = 79.002$ Pa s. A moment's reflection, however, proves that temperature fluctuations in the originally used system would have resulted in

Table 1. Particle and Fluid Parameters of the Numerical Trials to Reconstruct the OH-64 Data

Trial #	Amplitude of Motion, *m*	Frequency, 1/s	Maximum *Re*	Strouhal Number	Maximum *Ac*
1	0.025	0.3	0.07	133	28
2	0.1	1	1.13	39	2260
3	0.1	1	2.25	10	1130
4	0.1	0.2	0.09	1250	36
5	0.1	0.5	0.12	2000	920
6	0.05	2	5.63	0.798	226
7	0.075	1.0472	0.038	238	6.10E + 06
8	0.1	0.1	0.006	1587	69
9	0.5	0.8	0.36	78	214

significant variations of viscosity and lesser variations of density. We consider that these property fluctuations would be one of the principal sources of experimental error in the original study. Although we recognize this as an experimental uncertainty, in our study we used the values as they are reported because our intent is to reconstruct the original experimental data as faithfully as it is possible.

The Strouhal number, *Sl*, which appears in Table 1, is a dimensionless expression of the characteristic time of the fluids oscillatory motion divided by the characteristic time of the particle. The Strouhal number is the parameter that is more commonly used in the oscillatory type of flow, such as the one in the original study. In this case, the inverse of the angular frequency of the fluid oscillations, ω, was used as the characteristic time of the fluid for the computation of the Strouhal numbers.

$$Sl = \frac{1}{\omega\tau_P} = \frac{9\mu}{8\omega R^2\rho} = \frac{9\mu}{16\pi f R^2\rho} \quad (9)$$

where f is the frequency of oscillation ($\omega = 2\pi f$), τ_P is the diffusion characteristic time at the length-scale of the particle ($\tau_P = 8R^2\rho/9\mu$). Since we know that the OH-64 equation applies to rather low frequencies,[7,15] frequencies of the order of 1 s^{-1} were used in the nine trials of the present study.

Following the method used in OH-64, we back-tracked their equations and computed the total instantaneous hydrodynamic force for 28 time instances in each trial set. Thus, we computed the total hydrodynamic force, F_i, on the oscillating particle for 252 time values, which is approximately equal to the number of data points in the original set of experimental data. Using the same expression for C_D as in the original study and the more accurate expression $\Delta_A = 1$ (or $C_A = 1/2$) a new value for the history force, $F_{H\mathrm{NEW}}$, was computed as follows:

$$F_{H\mathrm{NEW}} = F_i - C_D\pi R^2\frac{1}{2}\rho(u_i - v_i) - \Delta_A\frac{1}{2}m_f\left(\frac{Du_i}{Dt} - \frac{dv_i}{dt}\right) + \left[\Delta_A\frac{1}{2}m_f\left(\frac{Du_i}{Dt} - \frac{dv_i}{dt}\right) - 0.5 * \frac{4}{3}\pi R^3\rho\left(\frac{Du_i}{Dt} - \frac{dv_i}{dt}\right)\right] \quad (10)$$

Hence, the parameter, $\Delta_{H\mathrm{NEW}}$ was calculated for every time instance in the nine "trials" by the following expression:

$$\Delta_{H\mathrm{NEW}} = \frac{F_{H\mathrm{NEW}}}{R^2(\pi\rho\mu)^{1/2}\int_{t_0}^{t}\frac{\frac{d(u_i - v_i)}{dt'}}{\sqrt{t-t'}}dt'} \quad (11)$$

All the calculations were performed using the program Microsoft Excel in a personal computer.

Analysis of the Computed Data

Figure 1 shows the instantaneous values of the parameter Δ_H, as it was calculated from the original OH-64 correlation and as it was computed from Eq. 11 (the revised expression). The results pertain to the conditions identified as Trials 2 and 3 in Table 1 and are plotted as a function of the instantaneous Reynolds number, *Re*. It is observed that there is a significant difference in the data calculated by the two expressions, especially at the lower values of *Re*. It is also observed that the points computed by the revised expression, Eq. 11, exhibit less scatter than the original data, which use the *Ac*. Such observations lead to the conclusion that the Reynolds number is a better parameter to express the functional relationship of the history terms than the originally used acceleration number, *Ac*.

Figure 1 is typical of the results obtained and the trends observed from Eq. 11 for all the trial cases: It is observed that the original and the revised values of the history coefficient differ significantly for the entire range of *Re*. Also, it is observed that at low *Re*, the values of Δ_H are close to the 6 and, at higher *Re*, Δ_H approach asymptotically a value that is approximately equal to 3. The main difference in diagrams similar to the one of Figure 1 for the other sets of data is the approximate values of *Re* where the curve flattens and becomes almost asymptotic. This is an indication that another parameter is needed in the functional representation of Δ_H vs. *Re* for the better representation of the data. This parameter would be similar to a scaling factor. After several tests and calculations, it was decided that the best parameter to be used as the scaling factor is the Strouhal number, *Sl*, defined by Eq. 9. The acceleration number, *Ac*, which was used in the original expression did not appear to be a good parameter for the correlation, because its range in an oscillatory flow is from 0 to ∞. The very large values of *Ac* result in significant "spikes" in the computed values of the total hydrodynamic force in the vicinity times given by the expression: $\omega t = n\pi$, with n being an integer. An inspection of the original publication by OH-64 confirmed the absence of any "spikes" in the graphs of the experimental data. For this reason, *Sl*, which has a finite and relatively narrow range of values (0–2000), has been used instead of *Ac* in the revised expression.

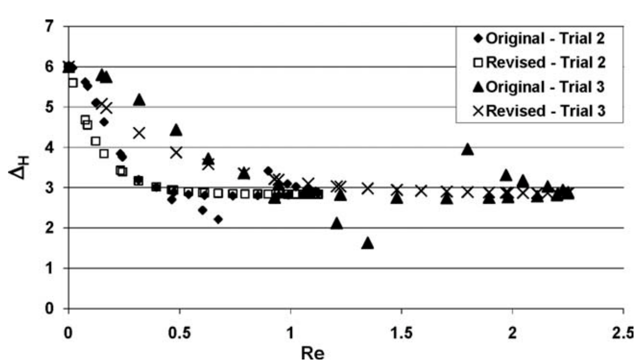

Figure 1. Relationship between original coefficient, Δ_H, and the revised expression, $\Delta_{H\mathrm{NEW}}$, as a function of *Re* for the Trials 2 and 3.

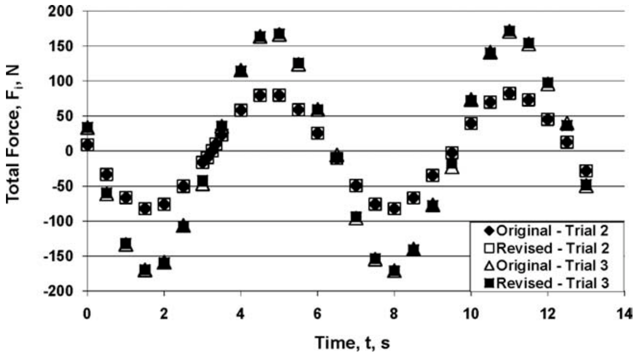

Figure 2. Total instantaneous hydrodynamic force vs. time for Trials 2 and 3 for the original and new computations.

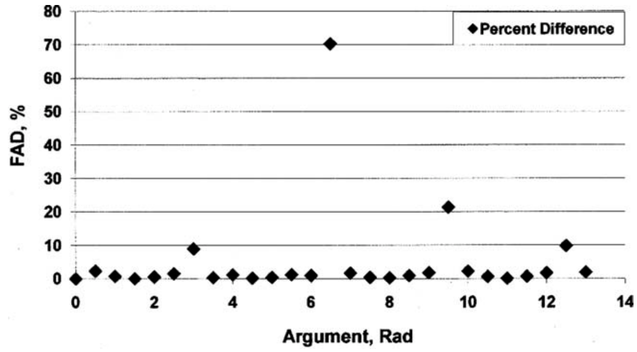

Figure 3. Absolute fractional difference of the total instantaneous hydrodynamic force resulting from the OH-64 expression and the new semi-empirical expression under the conditions of Trial 3.

With the aid of the software Table Curve 2D we used the computed values for $\Delta_{H\mathrm{NEW}}$, derived from Eq. 11, to derive a correlation $\Delta_{H\mathrm{NEW}} = f(Re, Sl)$. The best fit of the computed data resulted in the following correlation:

$$\Delta_{H\mathrm{NEW}} = 6.00 - 3.16\left[1 - \exp\left(-0.14\mathrm{Re}\, Sl^{0.82}\right)^{2.5}\right] \quad (12)$$

Accordingly, a semi-empirical transient equation of motion for solid spheres was put together using the OH-64 reconstructed data for the history term and the analytical and empirical equations that have been proven correct for the other terms. The Auton et al.[5] expression for the added mass term was also adopted. This new semi-empirical expression is as follows:

$$F_i = \frac{12}{\mathrm{Re}}\left(1 + 0.15\mathrm{Re}^{0.687}\right)\pi R^2 \rho (u_i - v_i)|u_i - v_i| + \frac{1}{2}\left(\frac{4}{3}\pi R^3 \rho\right)\left(\frac{Du_i}{Dt} - \frac{dv_i}{dt}\right) + \Delta_{H\mathrm{NEW}} R^2 (\pi\rho\mu)^{\frac{1}{2}} \int_{t_0}^{t} \frac{\frac{d(u_i - v_i)}{dt'}}{\sqrt{t - t'}} dt' \quad (13)$$

where the new coefficients of the history term, $\Delta_{H\mathrm{NEW}}$ are obtained from Eq. 12. Using this new correlation, several more trials were run to compare the total hydrodynamic force calculated using Eq. 2 to the total hydrodynamic force calculated using Eq. 13. We present these comparisons in the following paragraphs.

Results similar to the ones depicted in Figure 1 showed that there is a significant difference between the original and the new value of the history term coefficient. However, there is almost no difference in the results for the total instantaneous hydrodynamic force, F_i, that are computed using the original and the new expressions for the total transient force. This is shown in Figure 2, where the calculated results for the total hydrodynamic force that emanate from the two expressions are shown for the conditions identified as Trials 2 and 3 in Table 1. It is observed in Figure 2 that the results for the total transient hydrodynamic force from the original and the revised expression are almost identical over almost two cycles of the experimental data.

The relation seen in Figure 2 shows a negligible difference in the total hydrodynamic force that is exerted by the fluid on the solid sphere. This is an indication that the calculated terms for the new history term coefficient and the reconstructed relationship for the total hydrodynamic force, which is presented as Eq. 13, is faithful to the original data by Odar and Hamilton.[4] Thus, the new expression that is derived from these calculations, Eq. 13, is also faithful to the original data.

A good way to determine the agreement of two expressions is to calculate the fractional absolute difference, FAD, between them. In this case, the fractional absolute difference, FAD, is expressed as a percentage and defined as:

$$\mathrm{FAD} = 100 * \left|\frac{F_i - F_{i\mathrm{NEW}}}{F_i}\right| \quad (14)$$

Figure 3 depicts the FAD using data from Trial 3. It is apparent in this figure that the absolute fractional difference of the calculated results from the original and the new expressions is very low, typically <5%. This is certainly within the limits of the experimental error of the original data, which was of the order of 10%. It is also observed that the highest fractional difference in the computations is in the vicinity $\omega t = n\pi$ (n is an integer), where the acceleration term in the original expression, Ac, approaches infinity. This is another indication that Ac is not the best parameter to use in the correlation function of the transient hydrodynamic force.

Figure 4 depicts the values of the history term as calculated by the OH-64 correlation and by the revised expression

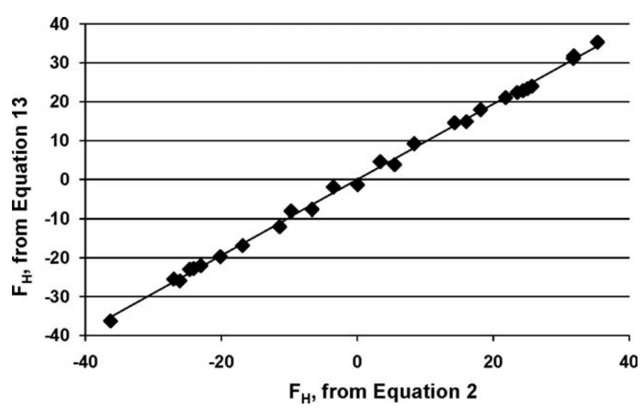

Figure 4. History force terms calculated by the OH-64 method and by Eq. 10.

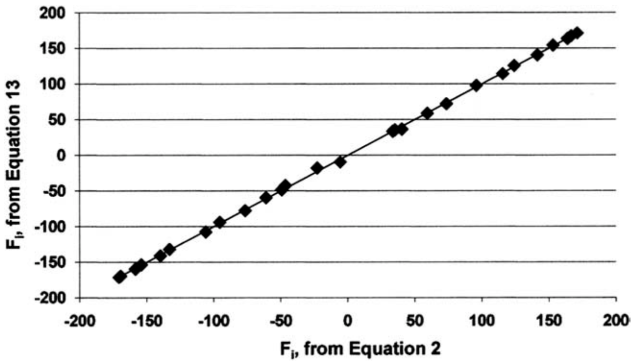

Figure 5. The direct relationship between the total hydrodynamic force using Eqs. 2 and 13. The slope of the line is equal to 1.

of Eq. 10 for the conditions of Trial 6. The solid line has slope equal to 1 and is the locus of equal values. It is observed that the most of the points of the graph are close to the line, but they do not lie on it. This implies that the calculated values for the history term from the two expressions do not differ a great deal and that for several of the conditions in this trial there is significant difference in values that are calculated from the two expressions. This difference is in general in the range 2–15%, but it extended to 60% in a few cases.

Although the values of the new correlation for the history term do not closely agree with the original correlations by OH-64, comparisons of the values for the total hydrodynamic force, F_i, prove that the two correlations give almost identical values. This is depicted in Figure 5, which shows the calculated total hydrodynamic force for the conditions of Trial 6. As in Figure 4, the solid line in this figure also has slope equal to 1 and represents the locus of the points of equal values. It is observed that all the points lie on this locus and this implies that there is very good agreement for the calculated total hydrodynamic force from the two methods by OH-64 and Eq. 13.

Conclusions

The OH-64 experimental data set is the most extensive and accurate set of experimental data on the transient hydrodynamic force on a solid sphere. However, the expressions derived in the original study suffer from the drawback of an erroneous assumption on the functional form of the added mass term. A set of the data for the total hydrodynamic force was reconstructed, following the computations of the original experimental study. From the reconstructed data, a new correlation for the history term was derived, which more accurately represents the currently used functional form of the added mass and the history terms. The analysis showed that the function of the history term is better correlated with the Reynolds and Strouhal numbers, rather than the acceleration number, which was used originally. The results show that total hydrodynamic force acting on a sphere accelerating in a viscous fluid may be accurately calculated by the new transient force semi-empirical correlation in the ranges of $0 < Re < 5$ and $0 < Sl < 2000$.

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Literature Cited

1. Boussinesq VJ. Sur la Resistance qu' Oppose un Liquide Indéfini en Repos… Comptes Rendu. *Acad Sci*. 1885;100:935–937.
2. Basset AB. *Treatise on Hydrodynamics*. London: Bell, 1888.
3. Schiller L, Nauman A. Uber die grundlegende Berechnung bei der Schwekraftaufbereitung Ver. *Deutch Ing*. 1933;44:318–320.
4. Odar F, Hamilton WS. Forces on a sphere accelerating in a viscous fluid. *J Fluid Mech*. 1964;18:302–314.
5. Auton TR, Hunt JRC, Prud'homme M. The force exerted on a body in inviscid unsteady non-uniform rotational flow. *J Fluid Mech*. 1988;197:241–257.
6. Odar F. Verification of the proposed equation for calculation of the forces on a sphere accelerating in a viscous fluid. *J Fluid Mech*. 1966;25:591–592.
7. Michaelides EE. Hydrodynamic force and heat/mass transfer from particles, bubbles and drops: the Freeman scholar lecture. *J Fluid Eng*. 2003;125:209–238.
8. Michaelides EE. *Particles, Bubbles and Drops—Their Motion, Heat and Mass Transfer*. New Jersey: World Scientific Publishers, 2006.
9. Faxen H. Der Widerstand gegen die Bewegung einer starren Kugel in einer zum den Flussigkeit, die zwischen zwei parallelen Ebenen Winden eingeschlossen ist. *Annalen der Physik*. 1922;68:89–119.
10. Maxey MR, Riley JJ. Equation of motion of a small rigid sphere in a non-uniform flow. *Phys Fluids*. 1983;26:883–889.
11. Michaelides EE. A novel way of computing the Basset term in unsteady multiphase flow computations. *Phys Fluids A*. 1992;4: 1579–1582.
12. Sano T. Unsteady flow past a sphere at low Reynolds number. *J Fluid Mech*. 1981;112:433–441.
13. Lovalenti PM, Brady JF. The hydrodynamic force on a rigid particle undergoing arbitrary time-dependent motion at small Reynolds numbers. *J Fluid Mech*. 1993;256:561–601.
14. Mei R, Adrian RJ. Flow past a sphere with an oscillation in the free-stream and unsteady drag at finite Reynolds number. *J Fluid Mech*. 1992;237:323–341.
15. Mei R. Flow due to an oscillating sphere and an expression for unsteady drag on the sphere at finite Reynolds numbers. *J Fluid Mech*. 1994;270:133–174.
16. Chang EJ, Maxey MR. Unsteady flow about a sphere at low to moderate Reynolds number. Part 1: Oscillatory motion. *J Fluid Mech*. 1994;277:347–379.
17. Kim I, Elghobashi S, Sirignano WA. On the equation for spherical-particle motion: effect of Reynolds and acceleration numbers. *J Fluid Mech*. 1998;367:221–253.
18. Tsuji Y, Kato N, Tanaka T. Experiments on the unsteady drag and wake of a sphere at high Reynolds numbers. *Int J Multiphase Flow*. 1991;17:343–354.
19. Poisson SD. *Memoire sur les Mouvements Simultanes d' un Pendule et de L' air Environnemant*. Mem. de l' Academie des Sciences, Paris. 1831;9:521–523.
20. Bataille J, Lance M, Marie JL. Bubbly turbulent shear flows. *ASME FED*. 1990;99:1–7.
21. Darwin C. A note on hydrodynamics. *Proc R Soc*. 1953;49:342–353.

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